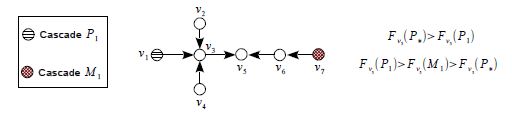
Homework 4

The Theory of social computing | Aadish Joshi

1. (2 points) Show that the objective function in the paper reading4-3 is neither submodular nor supermodular.



P1 is one positive cascade P1 and M1 is misinformation cascade

we deploy a new positive cascade P\* and assume the candidate seed set V \* is equal to V . Suppose that the probability on each edge is equal to 1, π(P1) = {v1} and π (M1) = {v7}, and the cascade priority at v3 and v5 is given as shown in the figure. We can observe that f({φ}) = 5, f({v2}) = f({v4}) = f({v2; v4}) = 4. Therefore, f({v2}) < f(φ); and f({v2}) + f({v4}) < f({v2} \ {v4}) + f({v2} ᴖ {v4}): This illustrates that inappropriately selecting positive seed nodes may lead to a wider spread of misinformation.

2 (2 points) Show that the objective function in the paper reading4-2 has a monotone nondecreasing submodular upper bound and a monotone nondecreasing submodular lower bound.

For a seed set of cascade , the objective function is to denote the expected number of the -active nodes when the diffusion process terminates. Given a budget and a candidate set , select a seed set for with such that is maximized. The objective function is is not a submodular function. However, in some special cases, the objective function is submodular.

The cascade priority is said to be homogeneous if each cascade has the same priority at each node. The cascade priority is said to be M-dominant if at each node, the priority of each misinformation cascade is higher than that of any positive cascade. The cascade priority is said to be P-dominant if at each node, the priority of each positive cascade is higher than that of any misinformation cascade. Then, we get two important results: (1) is monotone nondecreasing and submodular if the cascade priority is M-dominant or P-dominant. (2) is monotone nondecreasing and submodular if the cascade priority is homogeneous. Each cascade priority induces another two cascade priorities, defined as follows:

is a cascade priority at node induced by , satisfying (a) for each , if . (b) for each , if . (c) for each and , . is a cascade priority at node induced by , satisfying (a) and (b) above, and for each and , . For a seed set of cascade , we use (resp. ) to denote the expected -active nodes when each node replaces its cascade priority by (resp. ). Because is P-dominant and is M-dominant, and are both monotone nondecreasing and submodular. Furthermore, is an upper bound of and is a lower bound of . For each , .

3 (2 points) Show that any set function has a monotone nonincreasing supermodular upper bound and a monotone nonincreasing supermodular lower bound.

Consider following definition of the set function.

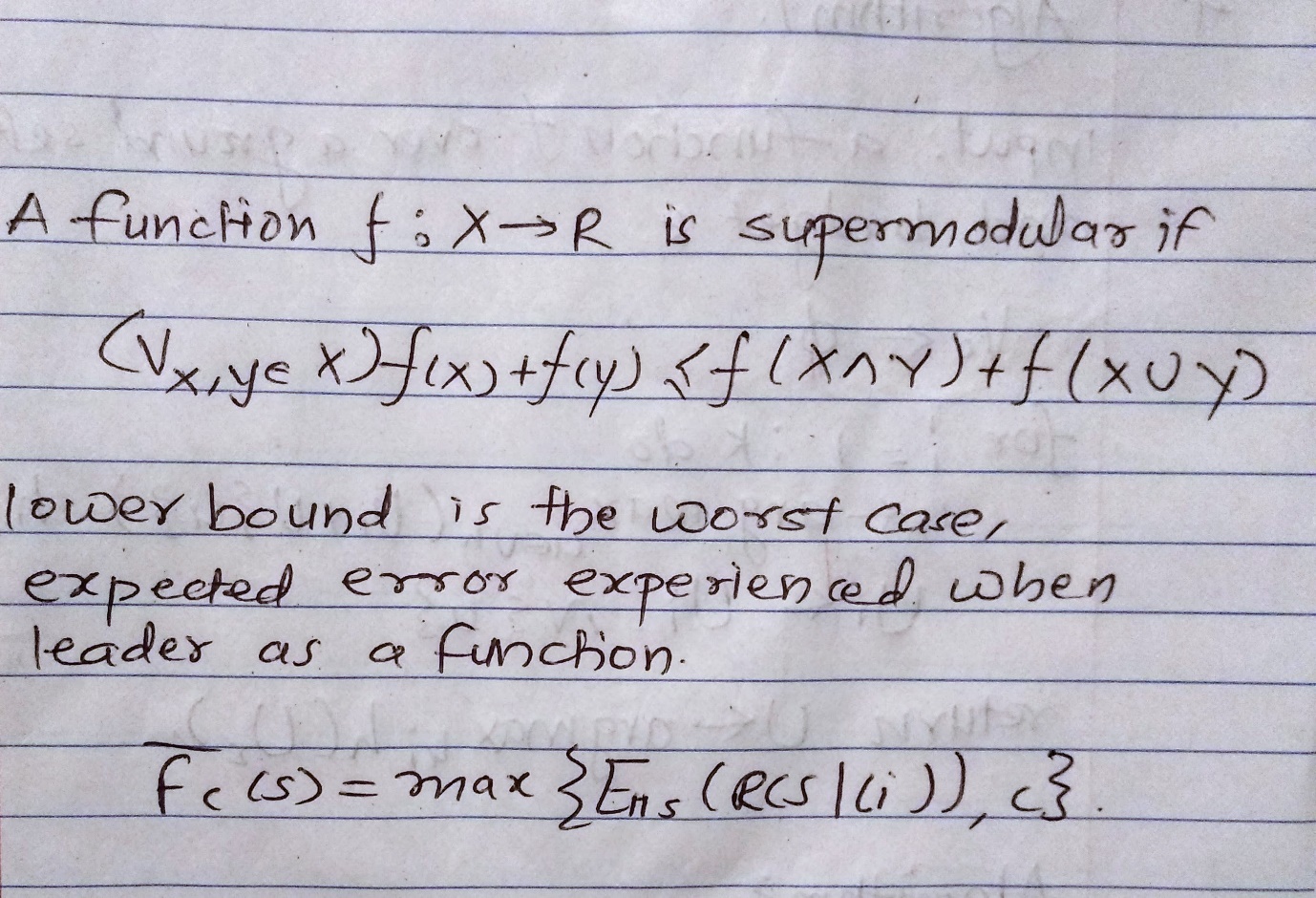
A set function f : 2V → R is called monotone increasing if f (X) ≤ f (Y ) for any X ⊆ Y .

A set function f : 2V → R is called submodular if f (X)+ f (Y ) ≥ f (X ∩Y )+ f (X ∪ Y ) for any X and Y ∈ 2V .

Lemma 2: γ () is monotone increasing and submodular.

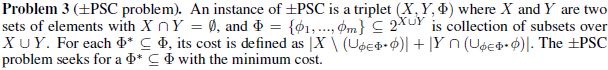
By Lemma 2, γ (Ai1 (i3, a)) − γ (Ai1 ) ≥ γ (Ai2 (i3, a)) − γ (Ai2), for any full-action A, 1 ≤ ii ≤ i2 ≤ i3 ≤ k and

a ∈ Ai3. Since γ- (S) = ∑A Pr [A|S]·γ (A).



4 (2 points) Can you give a DS decomposition to the objective function in the paper reading4-2?

Partial set cover problem is defined as follows.



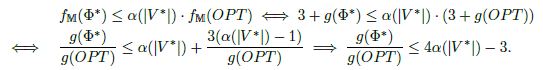
Where objective function is defined as follows.



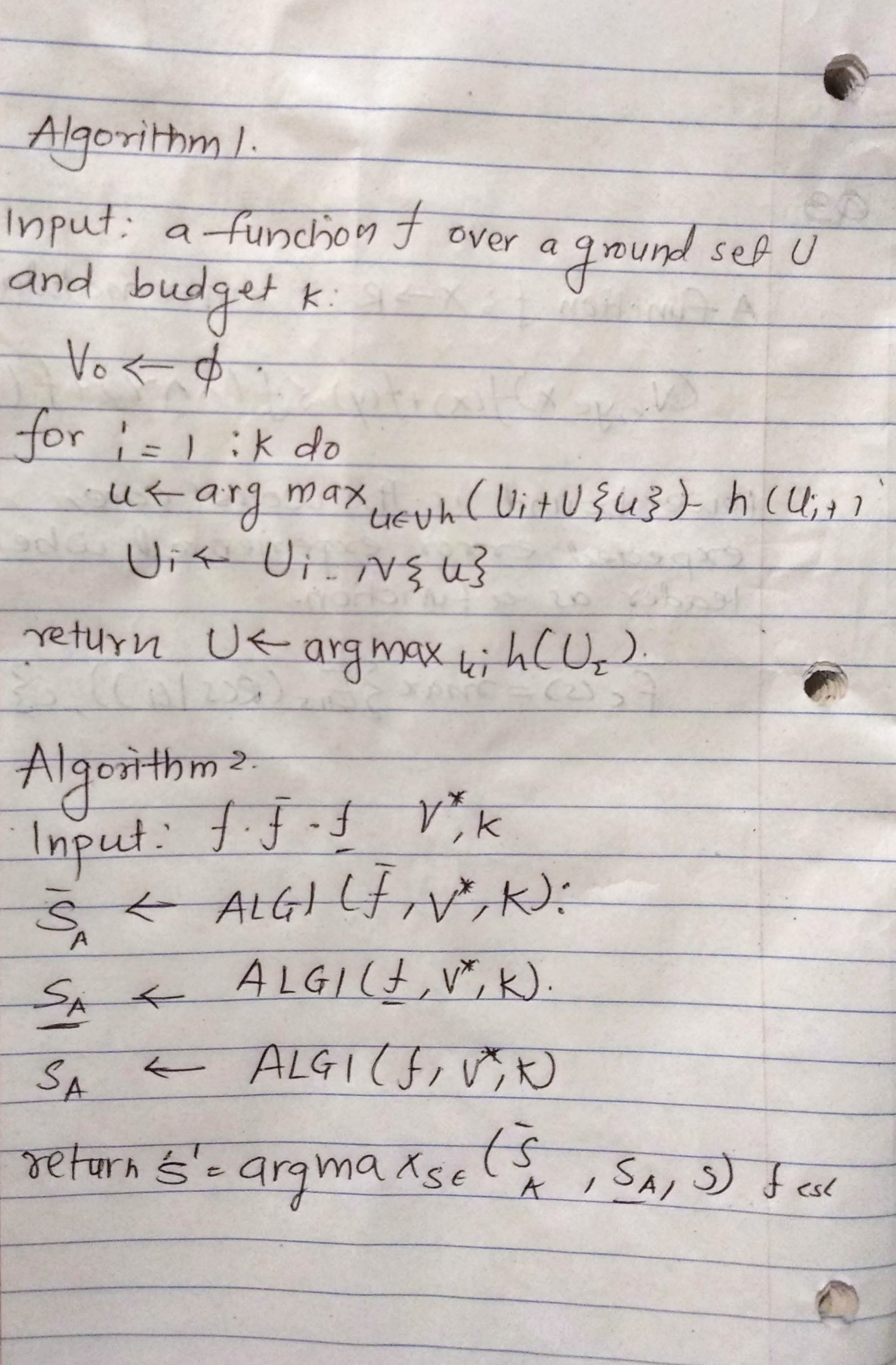
Also according to read4-2,



Suppose that φ\* is an (alpha|V \*|)-approximation to the Min-M problem for some   
(alpha|V\*|) > 1. We have



Since |V\*| = | φ | = m, φ\* is (4.alpha(m) -3) approximation to the instance of PSC problem.



5 (2 points) Design a method to apply the submodular-supermodular procedure to the maximization problem in the paper reading4-2 in case that you cannot find a DS decomposition for the objective function.

Every set function f: 2X -> R can be expressed as difference of two monotone nondecreasing submodular function g and h. i.e. f = g – h. where x is a finite set

According to the definition, any set function can be expressed as a DS function. That is, for set function h, there exist two submodular functions f and g such that h = f – g.

We select a submodular function g such that alpha(g) > 0.

And f(x) = h(x) + |alpha(h)| / alpha(g) . g(X). Then alpha(f) > 0

H = f – h = f - |alpha(h)| / alpha(g). g

We g(x) = sqrt(|X|)

Also we define, if alpha(h) >= 0, then h is submodular.

